RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. FIRST SEMESTER EXAMINATION, DECEMBER 2013

FIRST YEAR

Date : 14/12/2013 **PHYSICS (Honours)**

Time : 11 am – 3 pm Paper : I Full Marks : 100

[Use separate Answer Books for each group]

Group – A

(*Answer **any seven** questions taking at least **three** from each unit*)

Unit – I

1. a) Show that if A is an orthogonal matrix then Det A = ± 1. Explain the result.

   b) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -2 & -2 \end{bmatrix}$. Find the similarity transformation B, which diagonalises A. [(2+1)+(4+3)]

2. a) Find the general solution of $x^2 \frac{d^2y}{dx^2} - 6x\frac{dy}{dx} + 10y = 3x^4 + 6x^3$ given that $y = x^2$ and $y = x^5$ are linearly independent solutions of the corresponding homogeneous equation.

   b) Consider the equation $m\ddot{x} + K\dot{x} + Sx = F_0$ representing the motion of a weakly damped harmonic oscillator driven by a constant external force F_0. Solve the equation for small damping and sketch graphically the nature of variation of x with t. [6+(3+1)]

3. a) Find out the condition that the set of vectors $\overrightarrow{A_1}, \overrightarrow{A_2}, ..., \overrightarrow{A_n}$ are linearly independent. Show from this condition that the vectors $\hat{i}, \hat{j}$ and $\hat{k}$ are independent.

   b) Construct two orthonormalised vectors from the vectors $(2\hat{i} + 3\hat{j})$ and $(\hat{i} + 2\hat{j})$. Are the two vectors linearly independent? [5+5]

4. a) Consider a rotation about z-axis in the anticlockwise direction through an angle θ. How do the two vectors $\vec{A}$ and $\vec{B}$ change? How does $\vec{\nabla}\cdot\vec{A}$ change? How do $\vec{A}\cdot\vec{B}$ and $\vec{A}\times\vec{B}$ change? [2+2+4]

   b) Given that $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C}$. Can you say that $\vec{B} = \vec{C}$? What happens if $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C}$ and $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$? [2]

5. a) Find out the directional derivative of a scalar $\phi = 3x^2 + 4y^2 + 5z^2$ along the direction of the vector $\hat{i} + \hat{j} + \hat{k}$. In what direction is the derivative maximum?

   b) Find out $\vec{\nabla}\phi$ if $\phi = f(r)$. [(4+2)+4]

6. a) Verify Green's theorem in the plane for $\oint_C (2x - y^3)dx - xydy$, where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

   b) Derive an expression for $\vec{\nabla}\phi$ in orthogonal curvilinear co-ordinates. [6+4]

Unit – II

7. a) Using the Galilean transformation, show that in a two-body collision the law of conservation of linear momentum is Galilean invariant.

   b) A particle moves along a curve and has instantaneous polar coordinates (r, θ) given by $r = r_0 e^{\alpha t}, \theta = \omega t$ where α, ω are constant. Show that when $\alpha = \pm\omega$, the radial acceleration is zero, even though the radial velocity is non-zero. Explain. [5+5]

8. a) A particle moves in an elliptical path given by $\vec{r} = \hat{i}a\cos\omega t + \hat{j}2a\sin\omega t$. Find the angle between velocity and acceleration at time $t = \frac{\pi}{4\omega}$.

b) State and prove the work-energy theorem for a particle of mass m moving under the action of a force field, $\vec{F}$. If the force is conservative, show that the total mechanical energy is conserved.

c) A small block starts from rest from the top of a smooth sphere of radius R. Show that it loses contact with the sphere at a distance $\frac{R}{3}$ below the top. [3+4+3]

9. a) A uniform chain of total length 'a' is placed on a horizontal frictionless table so that the length 'b' of the chain dangles over the side. Show that the time t_0 required for the chain to slide off the table is given by $t_0 = \sqrt{\frac{a}{g}} \ln \frac{a+\sqrt{a^2-b^2}}{b}$.

b) A particle moves around a semi-circle of radius R from one end A of a diameter to the other end B. It is attracted towards its starting point A by a force proportional to its distance from A. It is also given that when the particle is at B, the force towards A is F_0. Show that the work done against the force when the particle moves around the semicircle from A to B is F_0R. [4+6]

10. a) Show that the general differential equation of motion of a body of variable mass, m(t), moving under a constant force, $\vec{F}$, is given by $\frac{d(m\vec{v})}{dt} = \vec{F} + \vec{u}\frac{dm}{dt}$ where $\vec{u}$ is the velocity of the added mass and $\vec{v}$ is the instantaneous velocity of the body, both with respect to the inertial frame.

b) Using the equation of motion in (a) obtain the equation of motion of rocket.

c) A rocket with initial mass m_i moves in a force free space with initial velocity v_i. Show that when the mass of the rocket is m_f, its velocity, v_f, is given by $v_f = C\ln\left(\frac{m_i}{m_f}\right)$ where C is the exhaust speed of the ejected gas.

Or,

A bomb moving with velocity $40\hat{i} + 50\hat{j} - 25\hat{k}$ m/s explodes into two pieces of mass ratio 1:4. The smaller piece goes out with velocity $200\hat{i} + 70\hat{j} + 15\hat{k}$ m/s. Deduce the velocity of the larger piece after the explosion. Also calculate the velocities of the pieces in the centre of mass reference frame. [4+2+4]

11. a) Consider the motion of two masses m_1 and m_2. Show that the kinetic energy of the particles in the centre of mass frame is equal to $\frac{1}{2}\mu|\dot{\vec{r}}|^2$ where $\vec{r}$ is the relative coordinate of m_1 with respect to m_2 and μ is the reduced mass given by $\mu = \frac{m_1m_2}{m_1+m_2}$

b) Show that for a system of particles the angular momentum about a point is equal to the angular momentum of a single particle of total mass, $M(=\Sigma m_i)$, situated at the centre of mass together with the angular momentum of the system of particles about the centre of mass.

c) Show that the mutually interacting forces on a system of particles have no effect on its total linear momentum. [3+4+3]

12. A mechanical harmonic oscillator of mass m and stiffness constant s is subjected to a viscous damping force proportional of its velocity, the coefficient of the damping force being R. The oscillator is being driven by a force $F = F_0\cos\omega t$. The steady state displacement is represented by $x = A\cos(\omega t - \delta)$.

a) Obtain the expressions for amplitude A and phase δ.

b) If ω_1 and ω_2 are the half power frequencies then find the frequency corresponding to velocity resonance.

c) Show that in the steady state, the time averaged input power equals the time averaged power dissipated through friction. [4+3+3]

Group – B

(*Answer **any three** questions*)

13. a) Define cardinal points of an optical system. Determine the system matric for a thick lens.
    b) Find the positions of cardinal points of a thick double convex lens in air where surfaces have radii of curvature 2cm and 4cm. The thickness of the lens is 2cm and refractive index 1·5. [(3+3)+4]

14. a) State Fermat's principle. Establish laws of refraction at spherical surface from Fermat's principle.
    b) Define three types of magnification and deduce a relation between linear and longitudinal magnification. [(1+4)+(3+2)]

15. a) Show that all rays parallel to axis of a parabolic mirror converge to its focus.
    b) The focal lengths of a Huygen's eyepiece are 2cm and 6 cm. Find the cardinal points of the eyepiece. Use matrix method and show that points in a diagram.
    c) Show that the nodal points and principal points of an optical system are identical if the medium on both sides are same. [3+(4+1)+2]

16. a) What do you mean by spherical aberration of an image formed by a lens? Mention a method to minimize this defect.
    b) What is chromatic aberration of a lens?
    c) Deduce an expression for axial chromatic aberration for non-parallel rays. [(2+2)+2+4]

17. a) What is apochromatic optical system?
    b) Deduce the achromatic condition for two lenses separated by a distance.
    c) An achromatic converging combination of focal length 30 cm is constructed with two lenses. If the dispersive power of one lens is two times that of the other, find the focal lengths of the two lenses.[2+5+3]